MTBF: Understanding Its Role in Reliability

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1.0: Introduction

The objective is to enable an individual who is unfamiliar with the use of the Mean Time Between Failures (MTBF) reliability parameter to understand its relationship in product predictions, failure rates, field performance, etc. After completing this tutorial, the participant will know and understand how MTBF relates to product performance over time.

In order for the practitioner to speak intelligently and authoritatively on the parameter, MTBF, it important at a minimum that a cursory understanding of the mathematical concepts involved with Reliability be mastered. Therefore, mathematical and practical treatments relative to MTBF are included in this tutorial using the exponential distribution model.

Additionally, to achieve a thorough understanding of statistics and reliability, many statistical experts list four kinds of understanding as shown below.*

1. Computational/Numerical
2 Visual/Graphical
3. Verbal/Interpretive

**For additional information on the items above, please contact the author of this tutorial.*

Introduction – cont'd

- **What is MTBF?**
  - Measure of rate of failure within the design life.
- **What is design life?**
  - Intended period of use which is expected to be failure free.

What do these terms mean?

- **Reliability?**
- **Failure?**
- **Failure Rate?**
- **Hazard Rate?**
- **MTBF/MTTF?**

Introduction – cont'd

Assumption of a constant failure rate

- When using Mean Time Between Failure (MTBF) or Mean Time to Failure (MTTF), a constant failure rate is assumed and the exponential distribution model prevails.
  - The exponential distribution is among one of the most common and useful life distribution models.
  - The exponential P.D.F occurs frequently in reliability engineering.
  - Describes the situation wherein the hazard rate is constant.
  - It is the distribution of time to failure t for a great number of electronic system parts.

Introduction – cont'd

Reliability Definitions

RELIABILITY [R(t)] - The probability that an item will perform its intended function without failure under stated conditions for a specified period of time.

FAILURE - The termination of the ability of an item to perform its required function as specified.

FAILURE RATE (FR) - The ratio of the number of failures within a sample to the cumulative operating time.

HAZARD RATE [λ, h(t)] - The "instantaneous" probability of failure of an item given that it has survived up until that time. Sometimes called the instantaneous failure rate.

2.0 Reliability Mathematics

What is the Probability Density Function (P.D.F.)?

Description of its meaning

Frequency distribution and cumulative distribution are calculated from sample measurements. Since samples are drawn from a population, the question is what can be said about the population? The typical procedure suggest a mathematical formula, which provides a ***theoretical*** model (p.d.f.) for describing the way the population values are distributed. ***A histogram and cumulative frequency functions*** are then estimates of these population models.

Reliability Mathematics – cont'd

P.D.F. – cont'd

- A positive continuous random variable follows an exponential distribution if the probability density function is as shown:

Thus the P.D.F.

$$f(x) = \begin{cases} ae^{-ax} & (For \ \ x \geq 0) \\ 0 & (For \ \ x < 0) \end{cases}$$

- It is important in reliability work because it has the same Central Limit Theorem relationship to Life Statistics as the Normal distribution has to Non-Life Statistics.

P.D.F. – cont'd

It is a probability density function (P.D.F.)

$$f(t) = \lambda e^{-\lambda t}$$

Lambda λ is a constant and is called the failure rate

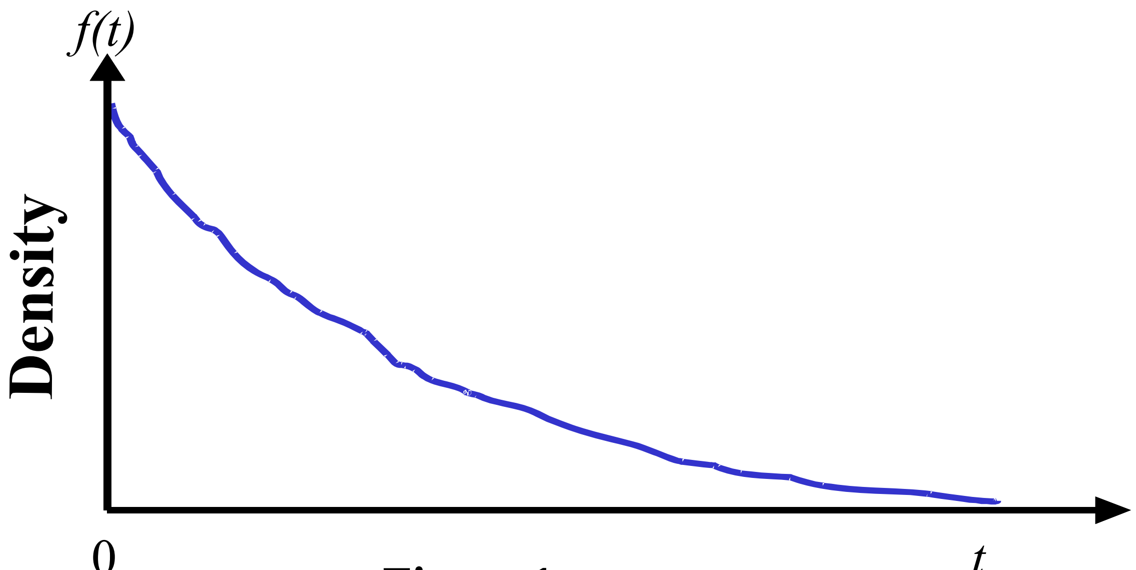


Figure 1

Reliability Mathematics – cont'd

What is the Cumulative Distribution Function (C.D.F.)?

- The cumulative distribution corresponds to a population model called cumulative distribution function C.D.F. and is donated by *F(t)*. It is related to the P.D.F. via the following relationship.

Use τ as a dummy variable; Let t = τ, then dt =dτ

$$F(t) = P(T \leq t) = \int_0^t f(\tau)d\tau$$

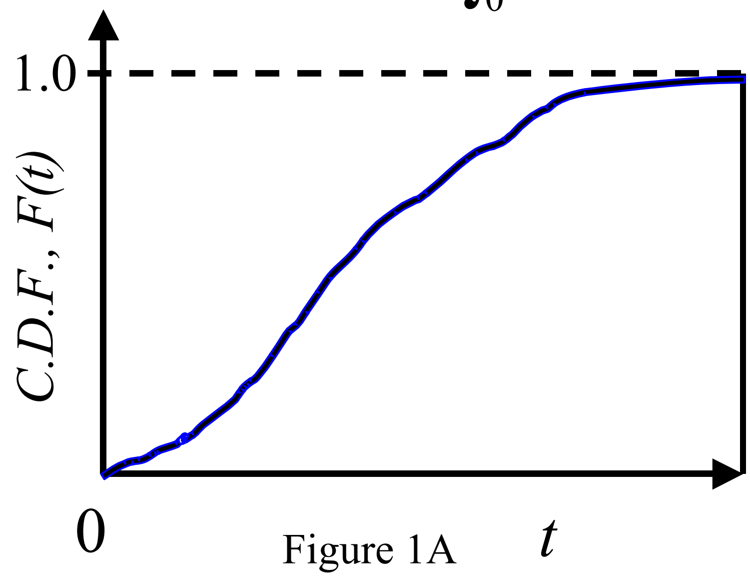


Figure 1A

C.D.F. Relationship to P.D.F.

- Reliability deals with failure times, *t*, which are nonnegative values. C.D.F. for population failures time is related to P.D.F. The P.D.F., which *f(t)* can be integrated to obtain the cumulative distribution function *F(t),* and the hazard function *h(t)* can be integrated to obtain the cumulative hazard function *H(t)*.

  - *The P.D.F. for the exponential distribution*

$$f(t) = \lambda e^{-\lambda t}$$

  - *The C.D.F for the exponential distribution*

$$F(t) = 1 - e^{-\lambda t}$$

Reliability Mathematics – cont'd

C.D.F. mathematical derivation

1. **Probability density function *(p.d.f.)***

$$f(t) = \lambda e^{-\lambda t}$$

2. C.D.F. is derived by integrating p.d.f.

$$F(\tau) = P(T \le \tau) = \int_0^t f(\tau)\,d\tau$$

$$F(\tau) = \int_0^t \lambda e^{-\lambda \tau}\,d\tau$$

$$F(\tau) = -e^{-\lambda \tau}\Big|_0^t$$

$$F(t) = 1 - e^{-\lambda t}$$

$$\therefore\ F(t) = 1 - R(t)$$

QED

$\therefore$ The probability that a component fails in the interval

$$0 \rightarrow t \quad is \quad F(t) = 1 - e^{-\lambda t}$$

Reliability Mathematics – cont'd

1. Hazard function

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

2. Cumulative hazard function:

(Integrating the hazard function to obtain the cum hazard function. Use dummy variable of integration τ)

$$H(\tau) = \int_0^t h(\tau)d\tau = \int_0^t \frac{f(\tau)}{R(\tau)}d\tau = \int_0^t \frac{\lambda e^{-\lambda\tau}}{e^{-\lambda\tau}}d\tau = \int_0^t \lambda d\tau = \lambda\Big|_0^t = \lambda t$$

$$\therefore H(t) = -\ln R(t)$$

$$Taking\ the\ natural\ \log of\ both\ sides : R(t) = e^{-\lambda t}$$

$$\ln R(t) = \ln e^{-\lambda t} = -\lambda t$$

$$\therefore \lambda t\ can\ be\ \text{expressed}\ as\ -\ln(e^{-\lambda t}) = -\ln R(t)$$

QED

Reliability Mathematics – cont'd

Hazard Rate

- The exponential P.D.F. is a valid useful life time to failure model for many debugged electronic components.

P.D.F. ⟶ $f(t) = \lambda e^{-\lambda t}$ $\quad 0 < t < \infty$

- Where λ represents a constant failure rate that does not vary with time. For reliability purposes, the C.D.F. is designated *F(t)* rather than *F(x) and* $F(t)t_2 - t_1$ is defined as the probability of failure in the interval $t_1 < T < t_2$.

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

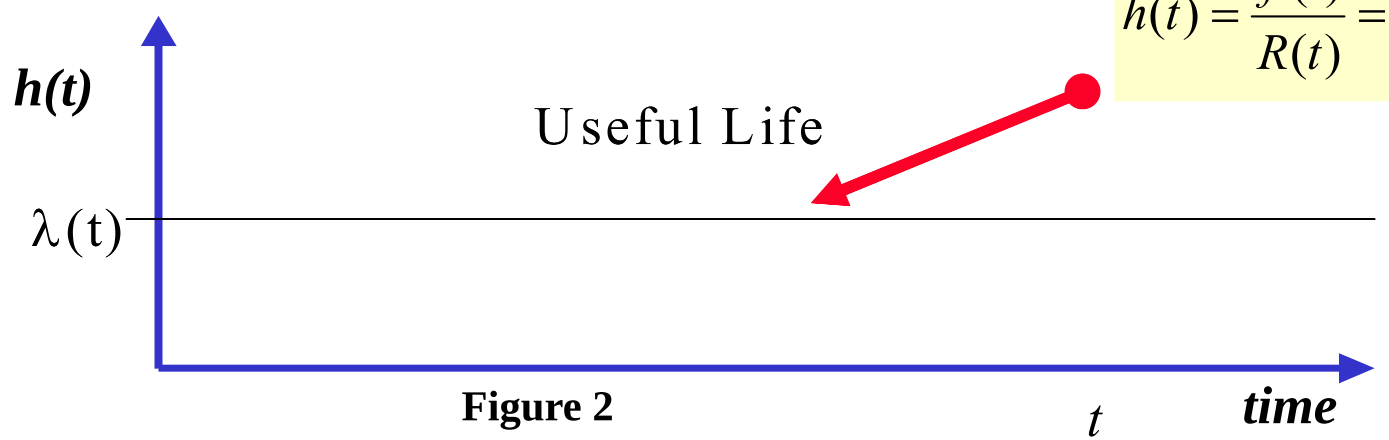


Figure 2

Reliability Mathematics – cont'd

Illustrations: Exponential

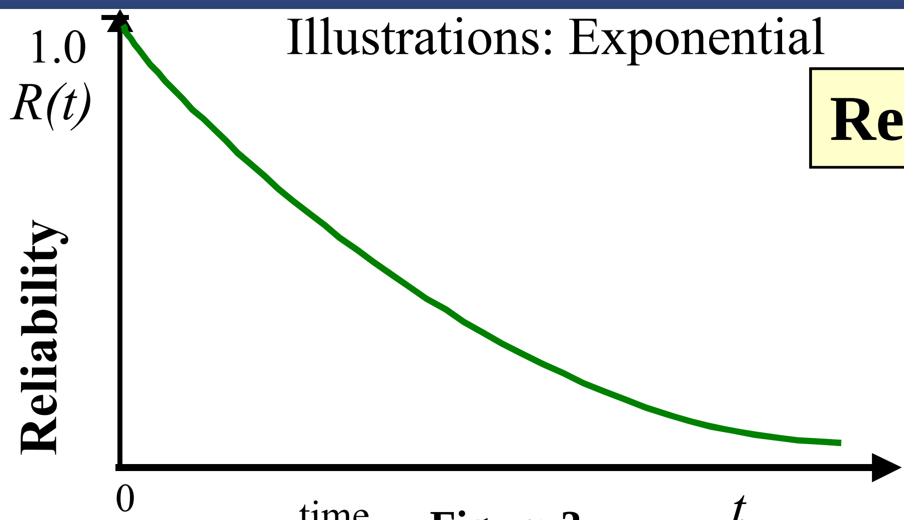


Figure 3

Reliability Curve

$$R(t) = e^{-\lambda t}$$

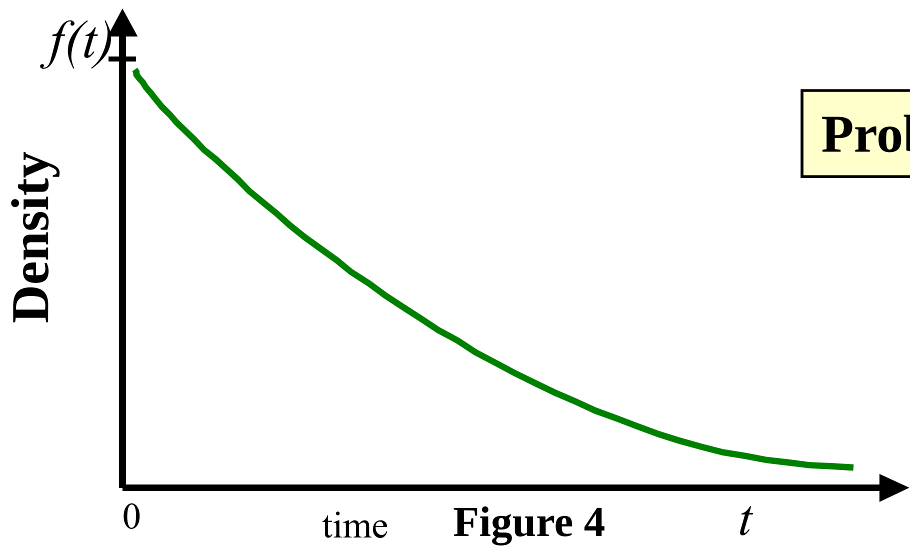


Figure 4

Probability Density Function Curve

$$f(t) = \lambda e^{-\lambda t}$$

Reliability Areas of Probabilities Illustration

$$F(t)=\int_0^t f(\tau)d\tau = P(T \le t)$$

Reliability exponential distribution plot

$$R(t)=1-\int_0^t f(\tau)d\tau$$

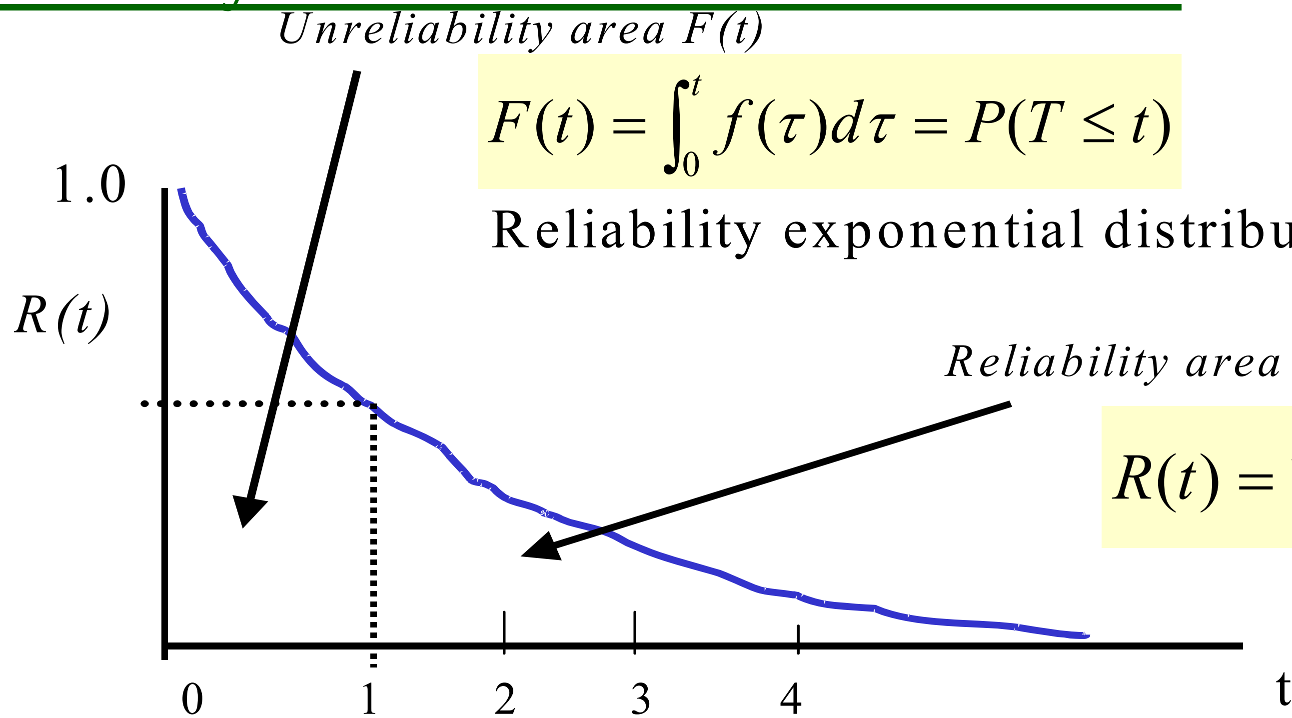


Figure 5

$$R(t) = 1 - F(t)$$

$$F(t) = -e^{-\lambda\tau}\Big|_0^t$$

$$R(t) = e^{-\lambda t}$$

$$\therefore F(t) = 1 - e^{-\lambda t}$$

Reliability Mathematics – cont'd

Example 1:

For exponential distribution, take the integral of *f(t), where*
$f(t) = \lambda e^{-\lambda t}$, *where t =1*

$$F(t) = \int_0^1 f(t)dt \qquad 0 \le t < 1$$

$$F(t) = \int_0^1 \lambda e^{-\lambda t} dt \qquad 0 \le t < 1$$

Solution

$$F(t) = \int_0^t \lambda e^{-\lambda t} dt = -e^{-\lambda t}\Big|_0^t = 1 - e^{-1} = 1 - 0.3678 = 0.6322$$

Reliability Mathematics – cont'd

Example 2:

An electronic device contains discrete transistors. Each transistor has a constant failure rate of $\lambda = 1\times10^{-5}$ failure rate/hour. What is the probability that a single transistor will survive a mission of 10^4 hours?

$$R(t) = 1 - F(t)$$

$$R(t) = e^{-\lambda t}$$

Solution

$$R(t) = e^{-\lambda t}$$

$$\therefore\ R(t) = e^{-\left(10^{-5}\right)\left(10^{4}\right)} = 0.905$$

$$F(t) = 1 - R(t) = 1 - 0.906 = 0.095$$

Reliability Mathematics – cont'd

Example 3:

What is the probability that it will survive a mission of 10^3 hours?

$$R(t) = e^{-\left(10^{-5}\right)\left(10^{3}\right)}$$

Solution

$$R(t) = e^{-0.01}$$

$$\therefore R(t) = 0.990$$

$$F(t) = 1 - R(t) = 1 - 0.990 = 0.010$$

3.0 Bathtub Curve

Reliability Bathtub curve for constant failure rate

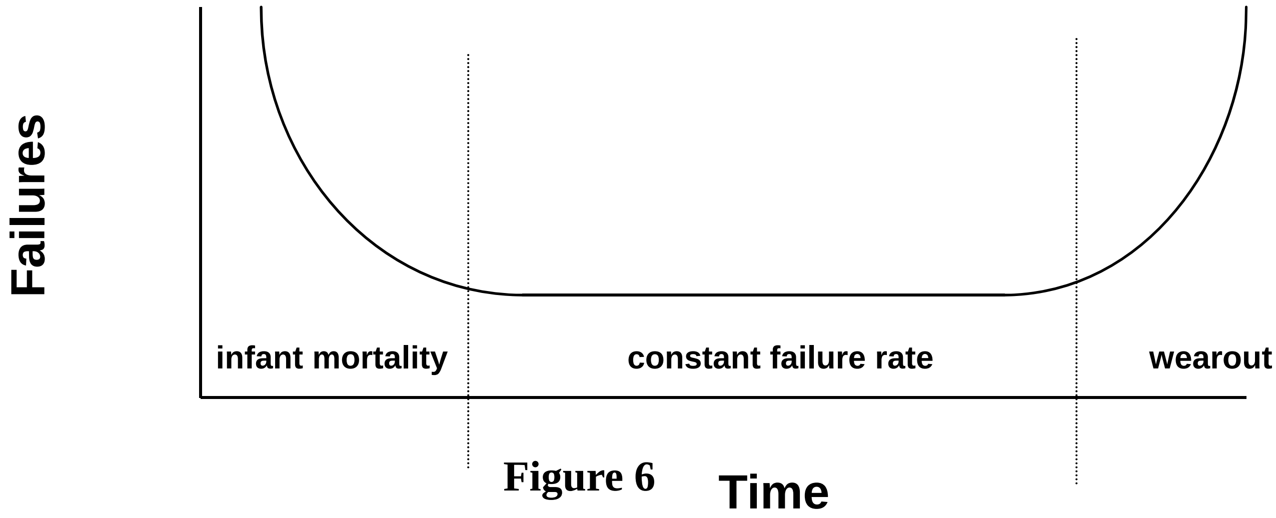


Figure 6

- Infant mortality- often due to manufacturing defects
- In electronics systems, there are models to predict MTBF for the constant failure rate period (Bellcore Model, MIL-HDBK-217F, others)
- Understanding wearout requires data on the particular device
  - Semiconductor devices should not show wearout except at long times
  - Electrical devices which wearout: relays, EL caps, fans, connectors, solder

Bathtub Curve – cont'd

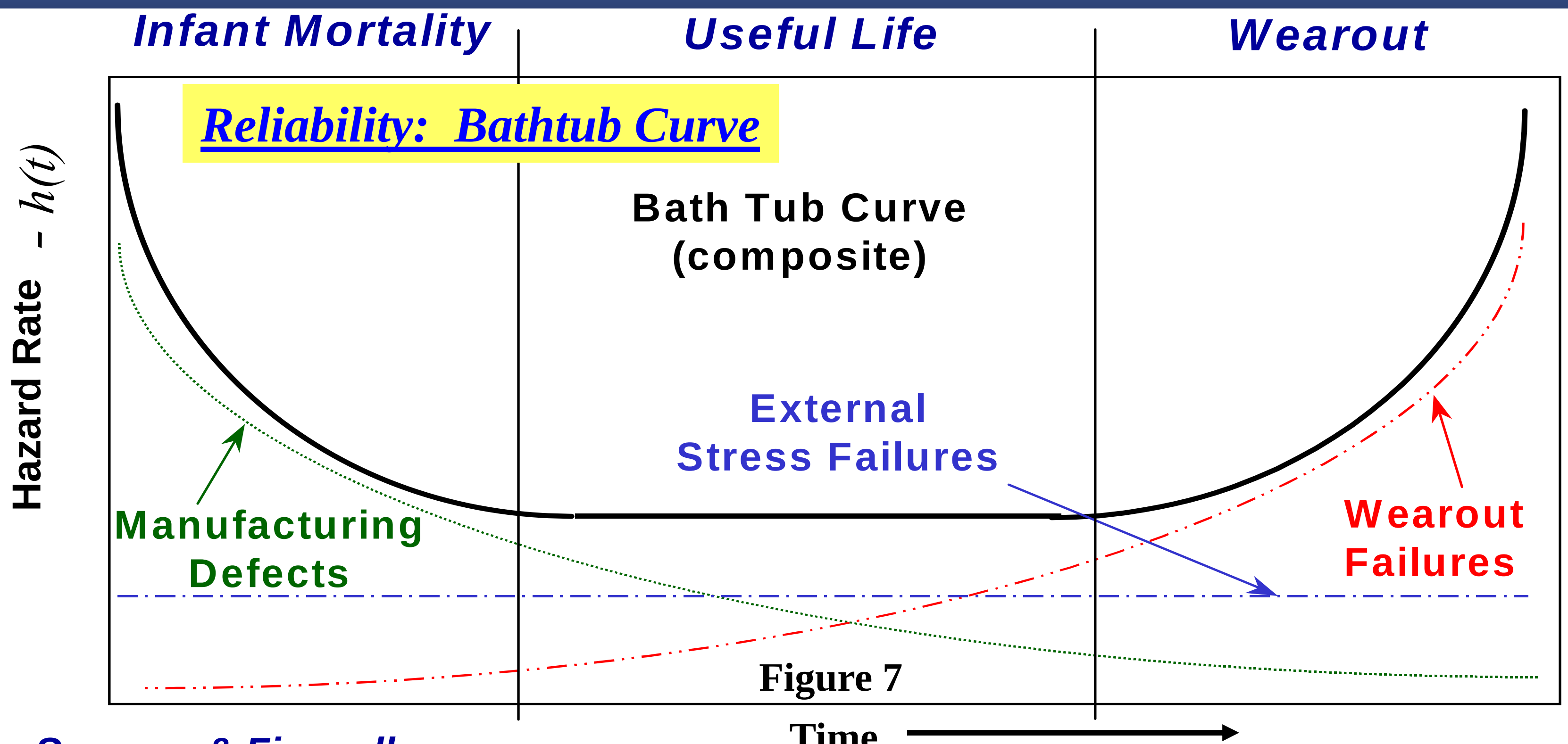


Figure 7

Sensors & Firewalls

- Analyzing short term warranty/RMA data
- Tying designs to mfg. capabilities
- Instituting process CTQ checkpoints
- Improving environment knowledge
- Reliability Metrics on Dashboards

Reliability Prediction & Validation

Beginning use of:

- Field & industry data
- Prediction tools
- Accelerated Life Testing

Wearout Mechanism Analysis

- Materials characterization
- Long-term data mining
- ALT (test to failure)
- System Life Modeling

4.0 Failure Rate

- **Graph: Cumulative distribution function (c.d.f.) for the exponential distribution function.**

Failure Rate
- Constant with respect to time
- An "average"

Hazard Rate
- A function of time
- "Instantaneous"

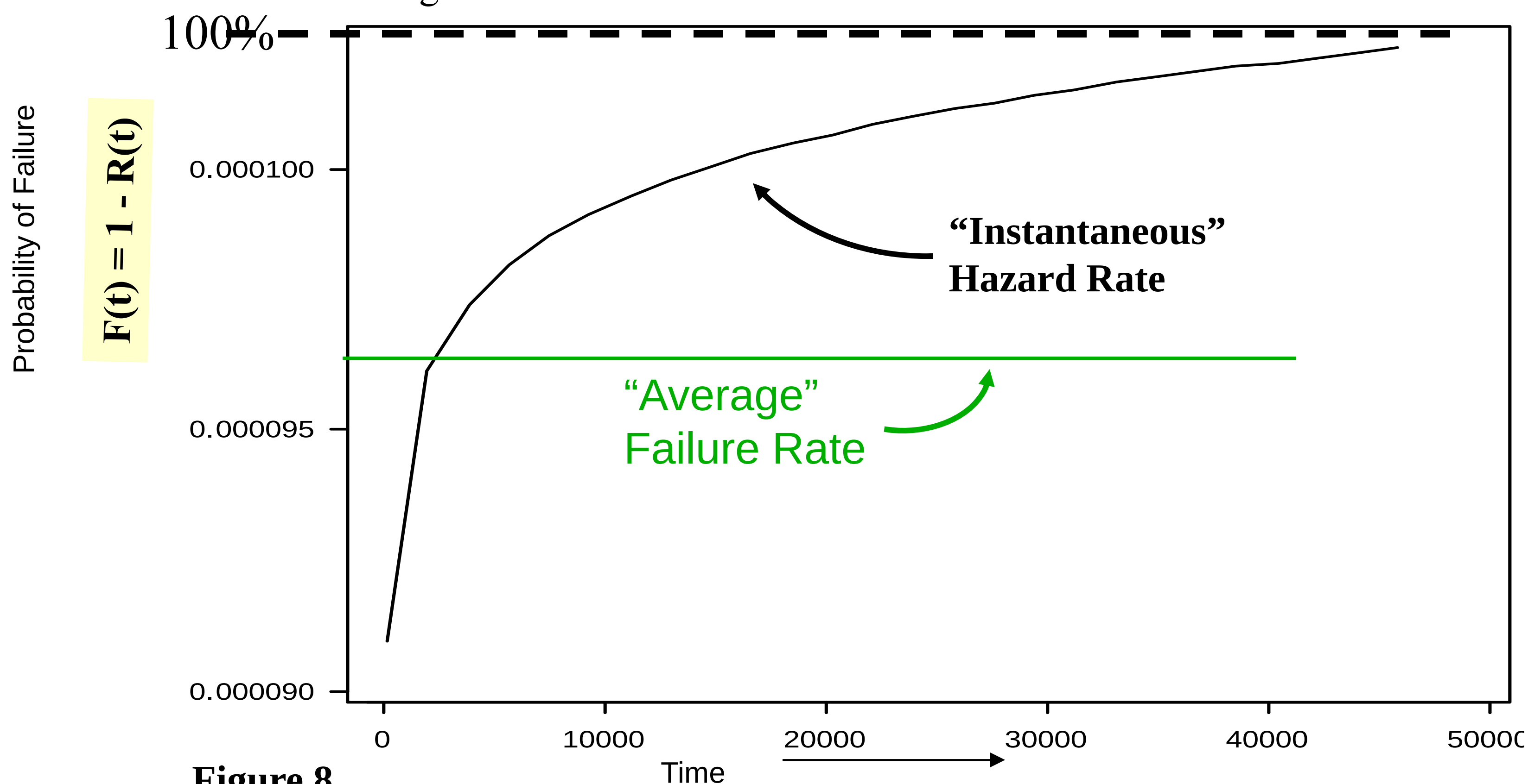


Figure 8

Reliability: Average Failure Rate vs. Hazard Rate

Failure Rate - cont'd

Average Failure Rate (AFR) for t_1, t_2

$$AFR = \frac{\int_{t_1}^{t_2} h(t)dt}{t_2 - t_1} = \frac{H(t_2) - H(t_1)}{t_2 - t_1} = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1}$$

Proof :

$$AFR = \frac{H(t_2) - H(t_1)}{t_2 - t_1}$$

If $H(t) = -\ln R(t)$

Then :

$$AFR = \frac{[-\ln R(t_2)] - [-\ln R(t_1)]}{t_2 - t_1} =$$

$$\therefore AFR = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1}$$

QED

Failure Rate - cont'd

Example 1.

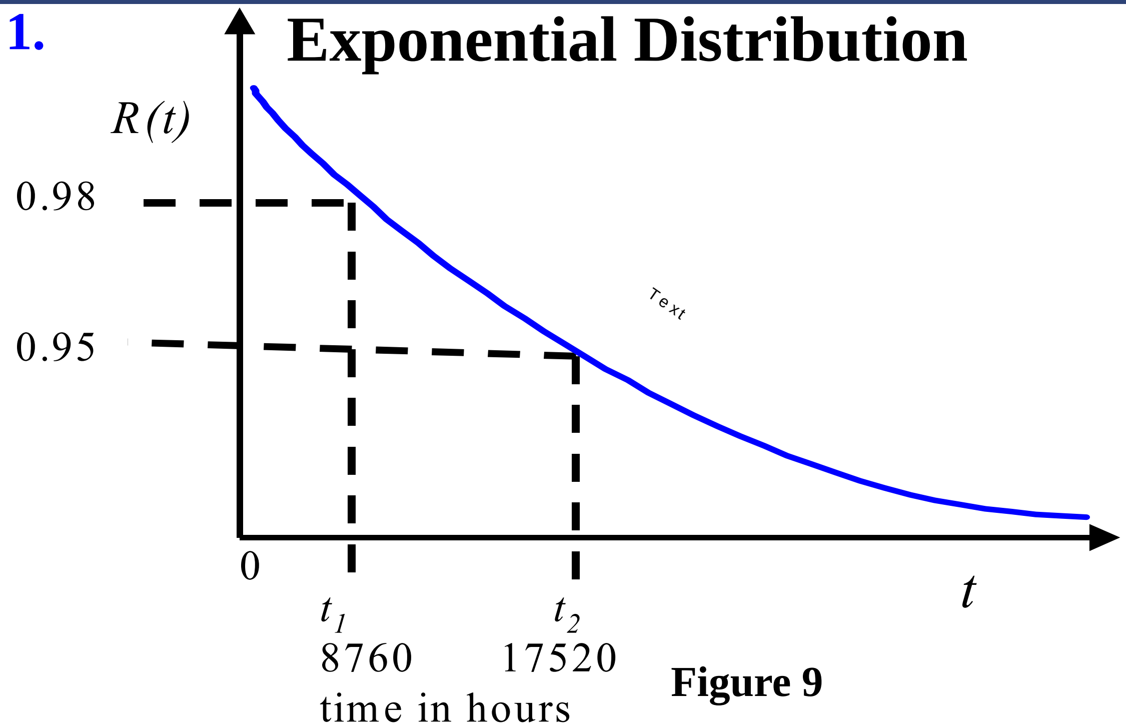


Figure 9

$$AFR = \frac{\ln R(t_1) - \ln R(t_2)}{t_2 - t_1}$$

$$AFR = \frac{\ln(0.98) - \ln(0.95)}{17520 - 8760} = \frac{-0.0202 - (-0.0513)}{8760}$$

$$AFR = \frac{0.0513 - 0.0202}{8760} = \frac{0.0311}{8760} = 0.00000355 = 3.55E-06$$

Example 1 continued

Illustration - Exponential Distribution

AFR can be represented by lambda (λ)

Therefore: λ = 3.55E-06, which is the hazard function *h(t)*

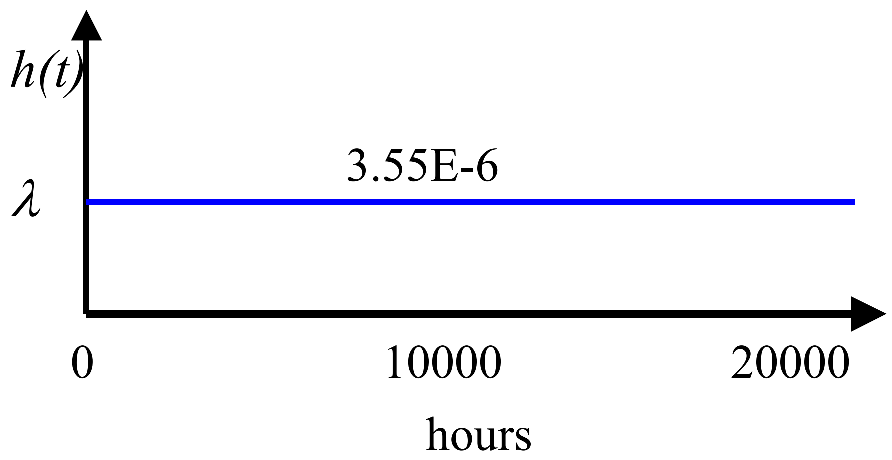


Figure 10

Failure Rate - cont'd

Estimating Failure Rate

- Lambda (λ) can be obtained as an estimate (λ -hat) of the true population for all operating hours for all units tested including failed and those that completed the test without failing.

  - It is the best estimate for complete or censored sample:

$$\hat{\lambda} = \frac{number\ \ of\ failures}{total\ \ unit\ \ test\ \ time}$$

  - The denominator is obtained by adding up all operating hours on test of all units tested, including those that failed and those that completing the test without failing.

Failure Rate - cont'd

Example 2: Five electronics Sub-systems failed from a sample of 1200 which were used constantly for 90 days. What is the Failure Rate?

Estimate of failure rate for λ

$$\hat{\lambda} = Failure\ rate = \frac{5\ failures}{1200*24*90} = \frac{5}{2{,}592{,}000}\ failures / hour$$

$$= 1.93E\text{-}06\ failures/hour$$

Other expressions

- Failures per million hours (Fpmh)

$$Fpmh = \lambda * 10E + 06 = (1.93E - 06)(10E + 06) = 1.93$$

- **Percent per thousand hours**

  This rate rate can be expressed by multiplying λ·(1E+05) resulting in the average failure rate = AFR = 0.193%/1000 hours

Failure Rate - cont'd

System failure rate

System failure function $h_s(t)$ is the sum of n component failure rate functions $h_1(t), h_2(t),\ldots,h_n(t)$. When the components have exponential lifetimes with parameters $\lambda_1, \lambda_2,\ldots, \lambda_n$, then the system has a constant failure rate equal

$$\lambda_s = \sum_{i=1}^{n} \lambda_i$$

Failure Rate - cont'd

➢ **Factors to convert h(t) and the AFR PPM or FIT when period is in hours.**

- Failure rate in %/K = [10E+5][(h(t)]
- AFR in %/K = [10E+5][AFR(T_1,T_2)]
- Failure rate in FITS = [10E+4][failure rate in %/K]
- AFR in FITS = [10E-9][AFR(T_1,T_2)]

5.0 MTBF

MTBF is not Life

Design Life: Intended period of use which is expected to be failure free.

MTBF: Measure of rate of failure within the design life.

EXAMPLES:

Item	Design Life	MTBF
Contactor	15,000 cycles	55,000 cycles
Pushbutton	3 million op's	12 million op's
CPU-Panel	15 years	37 years

Do not confuse MTBF with Design Life of an item

MTBF – cont'd

Mathematical Proof

$$MTBF = \int_0^{\infty} t\lambda e^{-\lambda t} dt = \frac{1}{\lambda}$$

By substitution

$$let: y = \lambda t$$

$$t = \frac{y}{\lambda}$$

$$dt = dy \frac{1}{\lambda}$$

$$MTBF = \frac{1}{\lambda}\int_0^{\infty} y\lambda e^{-y} dy$$

Integrating by parts

$$MTBF = \frac{1}{\lambda}\left[-(y+1)e^{-y}\right]_0^{\infty}$$

$$\therefore MTBF = \frac{1}{\lambda}$$ **QED**

Integrating by parts solution:

$$\int udv = uv - \int vdu$$

let

$$u = y \;\&\; du = dy$$

$$v = e^{-y} \;\&\; dv = e^{-y} dv$$

$$\int ye^{-y} dv = ye^{-y} - \int e^{-y} dy$$

$$= ye^{-y} - e^{-y}$$

by factoring

$$\left[-(y+1)e^{-y}\right]$$

Recall: $e^{-\infty} = 0$

MTBF – cont'd

<u>Mean Time Between Failures</u> [MTBF] - For **a repairable item**, the ratio of the cumulative operating time to the number of failures for that item.

Example 1

- AFR can be represented by lambda (λ)
- Therefore: λ = 3.55E-06, which is the hazard function *h(t)*

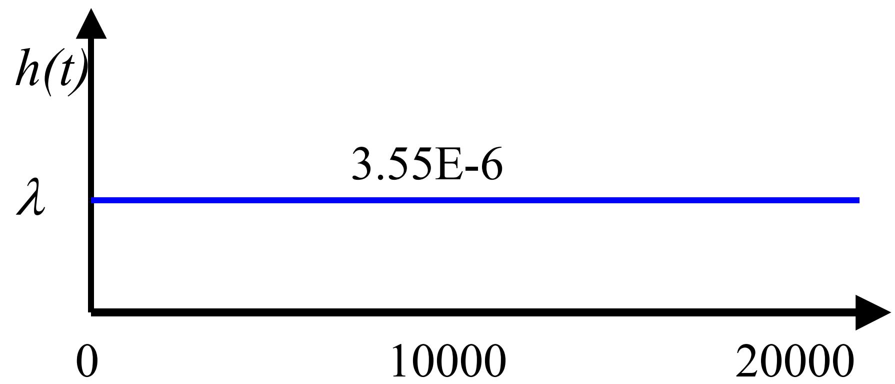


Figure 11

$$MTBF = \int_0^{\infty} t\lambda e^{-\lambda t}\,dt = \frac{1}{\lambda}$$

$$MTBF = \frac{1}{3.55E-6} = 281{,}757\,hours$$

MTBF – cont'd

Mean Time Between Failures (MTBF)

Example 2:

If a motor is repaired and returned to service six times during its life and provides 45,000 hours of service.

$$\lambda = \frac{failures}{time} = \frac{6}{45,000} = 0.00013333$$

$$MTBF = \frac{1}{\lambda} = \frac{1}{0.0001333} = 7,500\ hours$$

Also

$$MTBF = \frac{Total\ operating\ time}{\#\ of\ failures} = \frac{45,000}{6} = 7,500\ hours$$

MTBF – cont'd

Example 3

Using Chi-square model to find MTBF

MTBF Upper and lower bound is calculated such as 50%, 80% 90%, 95%, etc.

Data from example 2, previous page.

$$MTBF_{lower} = \frac{2T}{\chi^2_{\alpha,(\nu=2n)}}$$

$$MTBF_{upper} = \frac{2T}{\chi^2_{1-\alpha,(\nu=2n)}}$$

n ≡ Number of defects

T ≡ Total operating time

ν ≡ Degrees 0f freedom = 2n

α ≡ Significance level

MTBF – cont'd

Chi-square model - cont'd

Data from example 3, use 90% confidence or 10% level of significance.

Solution

lower bound

$$MTBF_{lower} = \frac{2T}{\chi^2_{\alpha,(v=2n)}}$$

$$MTBF_{lower} = \frac{2 * 45{,}000}{\chi^2_{0.10,\ v=2(6)}} = \frac{90{,}000}{\chi^2_{0.10,\ 12}} = \frac{90{,}000}{18.5} = 4{,}865 \text{ hours}$$

Can be found in any Chi-square distribution table

Upper bound

$$MTBF_{lupper} = \frac{2T}{\chi^2_{1-\alpha,(v=2n)}}$$

$$MTBF_{lupper} = \frac{2 * 45{,}000}{\chi^2_{1-0.10,v=2(6)}} = \frac{90{,}000}{\chi^2_{0.90,12}} = \frac{90{,}000}{6.3} = 14{,}286 \text{ hours}$$

MTBF – cont'd

Example 4

- **A**n electronics assembly has a goal of 0.99 reliability for one year.

  - ✓ What is the MTBF that the designer should work towards to meet the goal?

    - Reliability equation: $R(t) = e^{-\lambda t}$ and MTBF = $1/\lambda$
    - Solve for MTBF → $Hence: R(t) = e^{-t/MTBF}$

Solution:

$$\ln R(t) = \frac{-t}{MTBF}$$

$$-t = MTBF * \ln R(t)$$

$$MTBF = \frac{t}{-\ln R(t)} = \frac{8760}{-\ln 0.99} = 872{,}000 \text{ hours}$$

MTBF – cont'd

Example 5: If MTBF for an automobile is 100,000 miles...

Reliability as a Function of Mission Time t

Mission Length, t (miles)	Reliability*
1,000	99.0%
10,000	90.5%
50,000	60.7%
100,000	**36.8%**
200,000	13.5%

*Constant hazard rate

There is only a 36.8% chance of surviving past the period of one MTBF (i.e. when t = MTBF)

6.0 MTTF

<u>Mean Time To Failure</u> [MTTF] - For **non-repairable items**, the ratio of the cumulative operating time to the number of failures for a group of items.

Example 1: 1200 monitoring devices are operated for 90 days. During that time, five failures occur.

$$\lambda = \frac{failures}{time} = \frac{5}{1200*24*90} = \frac{5}{2{,}592{,}000} = 19.29E-07$$

$$MTTF = \frac{1}{\lambda} = \frac{1}{19.29E-07} = 518{,}403\ hours$$

Also,

$$MTTF = \frac{Total\ Operating\ Time}{Total\ Failures} = \frac{1200*24*90}{5} = 518{,}403\ hours$$

MTBF – cont'd

Example 2

- **A**n electronics assembly has a goal of 0.99 reliability for one year.
  - What is the MTTF that the designer should work towards to meet the goal?
    - ✓ Reliability equation: $R(t) = e^{-\lambda t}$ and MTTF = $1/\lambda$
    - ✓ Solving for MTTF

$$MTTF = \frac{t}{-\ln R(t)}$$

$$MTTF = \frac{8760}{-\ln R(0.99)}$$

$$MTTF = \frac{8760}{-(-0.01005)} = 871{,}642 \text{ hours}$$

7.0: MTTR

Mean Time To Repair (MTTR) - this is corrective maintenance, which includes all actions to return a system from a failed to an operating or available state. It is difficult to plan.

It entails, for example:

1. Preparation time: finding a person for the job, travel, obtaining tools and test equipment, etc.
2. Active maintenance time, i.e., doing the job
3. Delay time (logistic time: waiting for spare parts., once the job has been started.

$$MTTR = \frac{\sum(\lambda t_r)}{\sum \lambda}$$

Summation of expected times of individual failures modes

Summation of individual failure rates

$$Availability = \frac{MTBF}{MTBF + MTTR + mean\ preventive\ maintenance\ time}$$

8.0 Relationships Summary

Reliability Parameters

Description	Hours	Reliability	C.D.F. Unreliability	Hazard rate (failure rate/hour)	P.D.F.	Avg. failure rate (AFR)	Failure rate in PPM	Fails in time	Failures per million hours
Product	MTBF	*R(t)*	*F(t)*	*h(t)*	*f(t)*	% / K hrs.	PPM / K hrs.	FIT	Fpmh
AMPS-24	616860	0.9859	0.01410	1.6211E-06	1.5983E-06	0.1621	1621	1621	1.6211
CPU-3030	325776	0.9735	0.02653	3.0696E-06	2.9882E-06	0.3070	3070	3070	3.0696
CPU-640	307754	0.9719	0.02806	3.2493E-06	3.1582E-06	0.3249	3249	3249	3.2493
NCM-W	684196	0.9873	0.01272	1.4616E-06	1.4430E-06	0.1462	1462	1462	1.4616
NCA	476949	0.9818	0.01820	2.0967E-06	2.0585E-06	0.2097	2097	2097	2.0967

Relationships Summary - cont'd

Relationships – cont'd

I. Definitions:

Failure rate

The ratio of the number of failures within a sample to the cumulative operating time.

Example: PPM / K hrs.

MTBF

Mean time between failure, which means that 63.2% of the product would have failed by this time.

Example:

$$MTBF = \int_{o}^{\infty} t f(t)dt = \int_{o}^{\infty} t\lambda e^{-\lambda t} dt$$

Reliability *R(t)*

The probability that an item will perform its intended function without failure under stated conditions for a specified period of time.

Example:

$$R(t) = 1 - \int_{0}^{t} f(t)dt$$

Unreliability *F(t) - Commonly referred to as cumulative distribution function (CDF)*

The probability that an item will not perform its intended function without failure under stated conditions for a specified period of time. Also, commonly referred to as cumulative density function.

Example:

$$F(t) = \int_{o}^{t} f(t)\,dt$$

Hazard rate *h(t)*

The "instantaneous" probability of failure of an item given that it has survived up until that time. Sometimes called the instantaneous failure rate. It is the failure rate per unit time.

Example: 2.8E-6 / hour,

$$h(t) = \frac{f(t)}{R(t)}$$

Relationships Summary - cont'd

Relationships – cont'd

Hazard rate *h(t)*

The "instantaneous" probability of failure of an item given that it has survived up until that time. Sometimes called the instantaneous failure rate. It is the failure rate per unit time.

Example: 2.8E-6 / hour, $h(t) = \frac{f(t)}{R(t)}$

Probability Density Function (PDF) Commonly referred to as *f(t)*

It is denoted by *f(t)* where t is a the variable of interested where *f(t)dt* is the fraction of failure times of the population occurring in the interval dt. Basically, it assumes a mathematical formula that provides a theoretical model describing the way the population values are distributed. The definite integral of its domain must equal 1.

Example: $f(t) = \lambda e^{-\lambda t}, \quad 0 \leq t < \infty$

II. Examples

1. (1% / 1000 hrs.)

One percent per thousand hours would mean an expected rate of 1 fail for each 100 units operating 1000 hours.
Example: 0.1280%/1000 hours means that 1280 failures each 1 million units operating for 1000 hours.

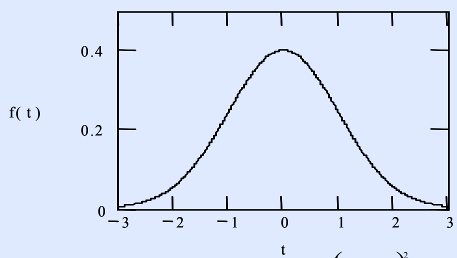
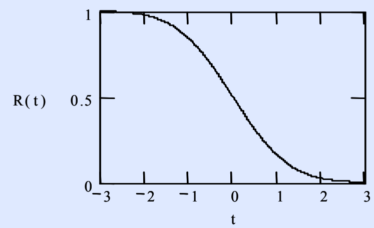
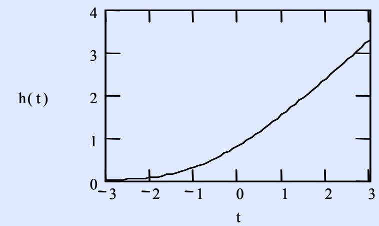
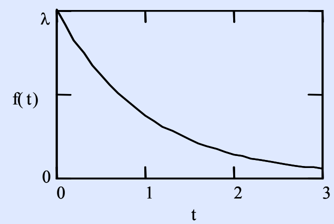
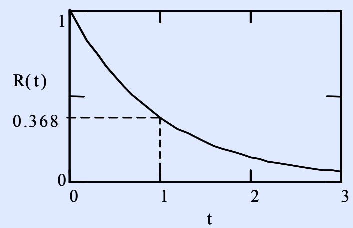
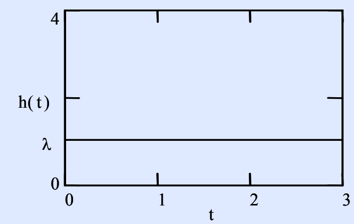
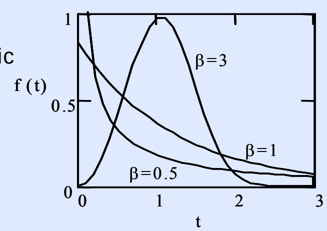
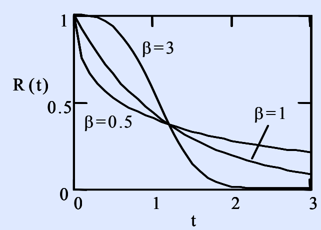
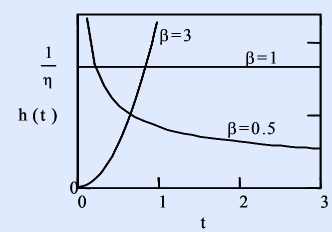
2. (PPM / 1000 hrs.)

One per million per thousand hours means 1 fail is expected out of 1 million components operating for 1000 hours.
Example: 1280 parts per million per thousand hours means that 1280 failures are expected out of 1 million components operating

3. Failure in time (FIT): 1 FIT = 1 Failure in One Billion Hours

Example: 1280 FITS = 1280 failures in one billion hours

Reliability Models

Type of Distribution	Parameters	Probability density function, f(t)	Reliability function, R(t) = 1- F(t)	Hazard function (instantaneous failure rate), h(t) = f(t) / R(t)
Normal Numerous applications. Useful when it is equally likely that readings will fall above or below the average.	Mean, μ Standard deviation, σ	 $f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\pi^2}}$	 $R(t) = \int_t^\infty f(t)dt$	 $h(t) = \frac{f(t)}{R(t)}$
Exponential Describes constant Failure rate conditions. Applies for the useful Life cycle of many Products. Frequently, time(t) Is used for x.	Failure rate, λ MTBF, θ $\theta = \lambda^{-1}$	 $f(t) = \lambda e^{-\lambda t}$	 $R(t) = e^{-\lambda t}$	 $h(t) = \lambda = \frac{1}{\theta}$
Weibull Used for many reliability applications. Can test for the end infant mortality period. Can also describe the normal and exponential distributions.	Shape, β Scale (characteristic life), θ Location (minimum life), γ Curves shown for γ = 0	 $f(t) := \frac{\beta}{\theta^\beta}\cdot(t-\gamma)^{\beta-1}\cdot\exp\left[\left[-\left(\frac{t-\gamma}{\theta}\right)\right]^\beta\right]$	 $R(t) := \exp\left[-\left(\frac{t-\gamma}{\theta}\right)^\beta\right]$	 $h(t) := \frac{\beta\cdot(t-\gamma)^{\beta-1}}{\theta^\beta}$

Reliability Models - cont'd

Type of Distribution	Parameters	Probability density function, f(t)	Reliability function, R(t) = 1- F(t)	Hazard function (instantaneous failure rate), h(t) = f(t) / R(t)

Gamma

Describes a situation when partial failures can exist. Used to describe random variables bounded at one end. The partial failures can be described as sub failures. Is an appropriate model for the time required for a total of exactly ? independent events to take place if events occur at a constant rate λ.

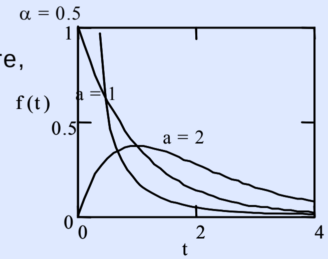
Failure rate, λ

Events per failure, or Time to ath failure

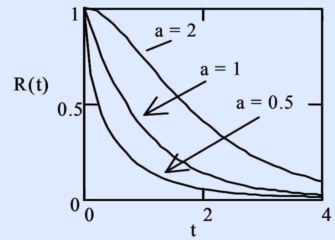
$$\mu = \frac{\alpha}{\lambda}$$

$$\sigma = \frac{\alpha^{\frac{1}{2}}}{\lambda}$$

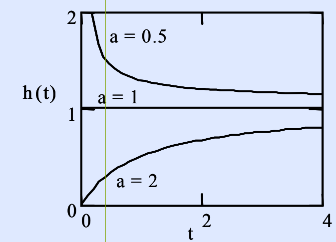
Note: when a is an integer I□(α) = (α−1)!



$$f(t) = \frac{\lambda}{\Gamma(\alpha)}(\lambda t)^{\alpha-1} e^{-\lambda t})$$



$$R(t) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)}\int_t^{\infty} t^{\alpha-1}(\lambda t)^{\alpha-1} e^{-\lambda t})dt$$



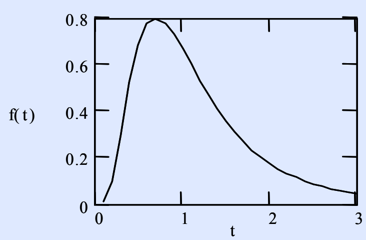
$$h(t) := \frac{f(t)}{R(t)}$$

Lognormal

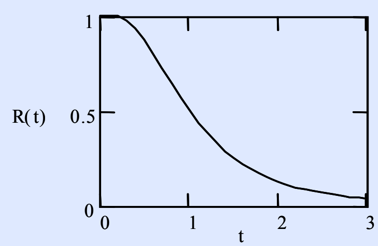
The Lognormal distribution is often a good model for times to failure when failures are caused by fatigue cracks. Let T be a random variable with a Lognormal distribution. By definition the new random variable X = ln T will have a normal distribution.

Mean, μ

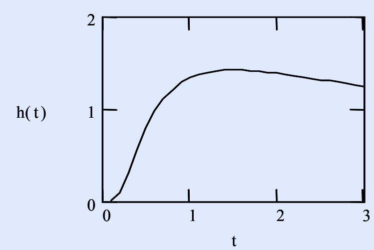
Standard deviation, σ



$$f(t) := \frac{1}{\sigma \cdot t \cdot (2 \cdot \pi)^{0.5}} \cdot \exp\left(\frac{-\ln(t-\mu)^2}{2 \cdot \sigma^2}\right)$$



$$R(t) := \int_t^{\infty} f(t)\,dt$$



$$h(t) := \frac{f(t)}{R(t)}$$

9.0 Summary

This paper did elaborate on the value of using the MTBF parameter. However, there have been tremendous improvements in solid-state devices over the years. In earlier times, electronic components were fragile, used glass tubes, filaments, etc., had inherent wear out mechanisms. By the same token, earlier solid-state devices had mechanisms that would cause failures in time, such as chemical contamination, metallization defects, and packaging defects, which resulted in corrosion and delaminating. Many of these defects were accelerated by high temperature, which resulted in successful use of the “burn-in” process to weed out “infant mortality." Statistical prediction during that period was valid and accepted because designs at that time consist mostly of discrete components; therefore, reliability statistical estimates of the life of a new design had for the most part, a reasonable correlation to the actual MTBF. Today hundreds of new components are introduced to the market almost every week and hundreds are probably taken off the market every week; therefore, it is impossible to make an accurate prediction based on a summation of parts reliability. Example: Mil-Std-217

Today’s components do not have wear-out modes that are within most electronics technologically useful life. Therefore, the vast majority of failures is due to defects in design or introduced in manufacturing. Unplanned events in manufacturing such as ECN, change in machine operators, or change in vendor’s capabilities of design, introduction of cost reduced parts, etc.; any of these or combinations can introduce a decrease in design margins. Hence: this affects reliability and increase field returns. Many experts feel that it is best to spend time, not on statistical predictions rather on discovering the real capabilities and identifying the weak links in the design or manufacturing process, and improving them. This approach will help realize significant improvement in reliability. The end user environment is even more uncontrolled. The end-user will always push the limits; therefore, a robust design will have a higher survivor rate for these extremes.

References

References

1. Patrick D.T. O'Connor, Practical reliability Engineering, third edition
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2. Paul A. Tobias & David C. Trindade, Applied Reliability, second edition
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