Estimating Expected Warranty Returns Using a Homogeneous Poisson Process (HPP)

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Introduction

Objective

- To illustrate the use of a Humongous Poisson Process (HPP)* model to approximate expected field returns over a defined warranty period.

Data Source

- Warranty data from AS400 Database

Analysis Tools

- Excel

Product Example

- NFS-6400PCA

Reference:

- Applied Reliability, second edition by Dr. Paul Tobias

*HPP describes a sequence of independently and identically exponentially distribution with rate λ, i.e., a constant recurrence rate.

Basis for Method

❖ Theoretical Basis for Method

- The purpose of this report is to show illustrate the theoretical proof and use an example of of the NFS-6400 to approximate expected warranty returns..

- The Poisson is the basis for using such method. The Poisson model is a discrete distribution and the Weibull, exponential, Lognormal, etc., are continuous distributions. Mathematically speaking, discrete events occurring randomly in a continuum (time, cycles, etc.) cannot be represented by a single continuous distribution function. Expected returns on a monthly basis are discrete data. In simply terms this means that the failure rate is continuous for each month's interval and discontinuous at the of the break between months. Additionally, in order to use the Poisson model, the failure rate must be exponentially distributed; therefore, the data can be classified as piecewise exponential. Hence: failures are flowing past a fixed point monthly (discrete) and thus are discrete events and can be classified as stochastic point processes. This restriction is mathematically fine because this method uses a nonparametric approach in determining expected returns

- The appropriate model in the above paragraph is the Poisson distribution, which describes the situation when events occur randomly and at a constant failure rate. Therefore, the mathematical foundation for this method is based on a homogeneous Poisson process (HPP).

Basis for Method – cont'd

Theoretical Basis for method – cont'd

- As previously stated, HPP describes a sequence of independently and identically exponentially distribution with failure rate λ. That is, the probability of observing exactly N(t) = k replacements in interval (0, *t*) is

$$P[N(t)=k]=\frac{(\lambda t)^k e^{-\lambda t}}{k!},\ \ (\text{for } i=0,1,2,\ldots)$$

where λ is the mean rate of occurrence, thus λ*t* is the expected number of events or returns (*0*, *t*)

✓ Poisson's ***mean*** can represent the expected returns as (λt).

$$\frac{1}{\lambda}=\text{Mean Time Between Failures (MTBF)}$$

✓ Failures follows a stochastic process, which means random failures in a fixed interval or time.

Basis for Method – cont'd

Theoretical Basis for Method – cont'd

- The Cumulative distribution function (C.D.F.) can be expressed using the sum of Poisson probabilities as shown below.

$$F_k(t) = 1 - \sum_{i=0}^{k-1} \frac{(\lambda t)^i e^{-\lambda t}}{i!} = 1 - e^{-\lambda t} \sum_{i=0}^{k-1} \frac{(\lambda t)^i}{i!}$$

- Therefore, based on the HPP model, the following process for expected returns is supported.

$$\text{Expected returns} = \sum_{i=0}^{k} (\lambda t)$$

which is the Poisson distribution leading to $\mu = (\lambda t)$ =(failure rate · 1 month ·Sales)

- The method should give approximately the expected returns *(allowing for fails of replacements within a month, at the same rate as the fails on the original parts*).
  Hence: the process modeled by HPP with λ_1 for month 1 changing to an HPP with λ_2 for month two, etc. The process works better with the same lambda for each period.

Basis for Method – cont'd

Theoretical Basis for Method – cont'd

- If k is described as an independently distributed Poisson (n_k, λ_k) random variable where λ represents the piecewise exponential failure rate during the kth service period.

- Expected returns ~ Poisson (sales, failure rate)

The derivative of expected returns is equal to the returns rate.

$$M(t) = \lambda t$$

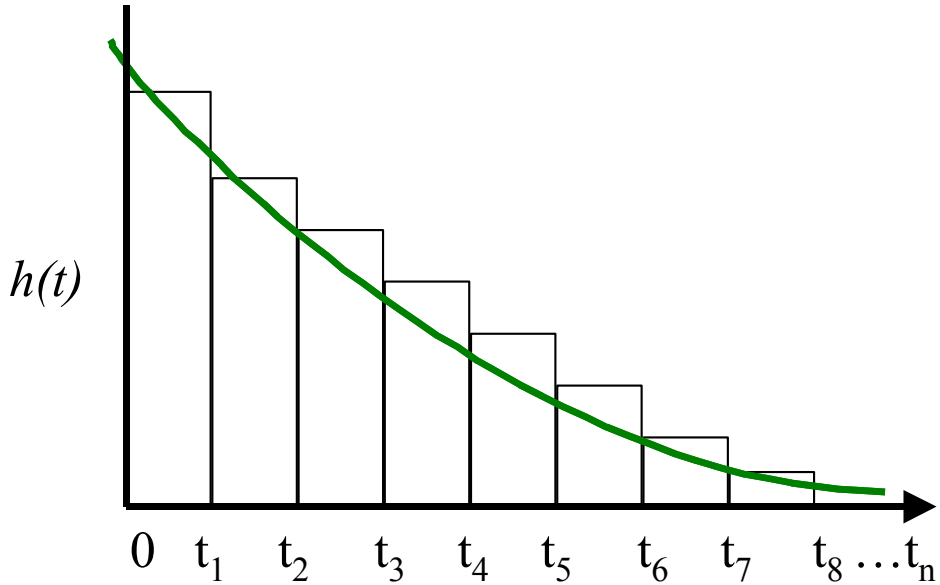
$$\frac{dM(t)}{dt} = \lambda = m(t)$$

$$\therefore m(t) = \lambda$$

$$\text{Expected value} = M(t) = \int_0^t m(t)dt = \int_0^t \lambda dt = \lambda t$$

Basis for Method – cont'd

Illustration: Piecewise Exponential Distribution



If the probability of the following exist,

$$P(t_1 < T < t_2) = \int_{t_1}^{t_2} f(t)dt \quad \text{where } T = \text{a continuous random variable}$$

then

$$\text{Example: Expected returns} = M(t) = \int_{t_1}^{t_2} \lambda dt = \lambda t_2 - \lambda t_1$$

Can be shown as,

$$\therefore \text{Expected returns} = \sum_{i=1}^{k} \lambda_k s_k \cdot 1$$

Basis for Method – cont'd

Theoretical Basis for Method – cont'd Calculation format

Expected returns→ Shipped↓	Month1	Month 2	Month3	Mnth4	Month5	Month6	Month7
Month1	$\lambda_1 S_1$	$\lambda_2 S_1$	$\lambda_3 S_1$	$\lambda_4 S_1$	$\lambda_5 S_1$	$\lambda_6 S_1$	$\lambda_7 S_1$
Month2		$\lambda_1 S_2$	$\lambda_2 S_2$	$\lambda_3 S_2$	$\lambda_4 S_2$	λS_2	$\lambda_6 S_2$
Month3			$\lambda_1 S_3$	$\lambda_2 S_3$	$\lambda_3 S_3$	$\lambda_4 S_3$	$\lambda_5 S_3$
Month4				$\lambda_1 S_4$	$\lambda_2 S_4$	$\lambda_3 S_4$	$\lambda_4 S_4$
Month5					$\lambda_1 S_5$	$\lambda_2 S_5$	$\lambda_3 S_5$
Month6						$\lambda_1 S_6$	$\lambda_2 S_6$
Month7							$\lambda_1 S_7$

Example

Expected returns month1: $\lambda_1 s_1$

Expected returns month2: $\lambda_2 S_1 + \lambda_1 S_2$

Expected returns month3: $\lambda_3 S_1 + \lambda_2 S_2 + \cdot \lambda_1 S_3$

Expected returns month4: $\lambda_4 S_1 + \lambda_3 S_2 + \lambda_2 S_3 + \lambda_1 S_4$

Expected returns month5: $\lambda_5 S_1 + \lambda_4 S_2 + \lambda_3 S_3 + \lambda_2 S_4 + \lambda_1 S_5$

Expected returns month6: $\lambda_6 S_1 + \lambda_5 S_2 + \lambda_4 S_3 + S_4 \lambda_3 S_4 + \lambda_2 S_5 + \lambda_1 S_6$

Expected returns month7: $\lambda_7 S_1 + \lambda_6 S_2 + \lambda_5 S_3 + \lambda_4 S_4 + \lambda_3 S_5 + \lambda_2 S_6 + \lambda_1 S_7$

Product: NFS-6400

NFS-64– cont'd

			Jan-04	Feb-04	Mar-04	Apr-04	May-04	Jun-04	Jul-04	
		Sales	P1	P2	P3	P4	P5	P6	P7	
Jan-04	S1	690	2	2	3	5	0	2	6	
Feb-04	S2	1051		1	2	7	7	3	2	
Mar-04	S3	855			0	5	2	5	1	
Apr-04	S4	773				2	0	4	7	
May-04	S5	880					0	1	7	
Jun-04	S6	962						0	2	
Jul-04	S7	901							0	
			2	3	5	19	9	15	25	
			688	1736	2586	3340	4211	5158	6034	**Avg. for all*** **
Piecewise exponential failure rates			0.002906977	0.001728	0.001933	0.005689	0.002137	0.002908	0.004143	0.002471795
			***1st month**					****6 mo. Avg.**		
								0.002884		

Legend:

*Used the actual piecewise exponential failure rate for the first month.

**Used the first 6 months of failure rates average for the 6th month's point. This will represent the piecewise exponential estimate for the discrete 6th month.

***Used the average failure rate values of all data in the input table to estimate the piecewise exponential failure rate for the 12th, 18th month, etc.

Note:

This is probably the best approach in order to comply with the Poisson model, which is the basis for the methodology used to estimate returns in the Reliability Analysis Workbook.

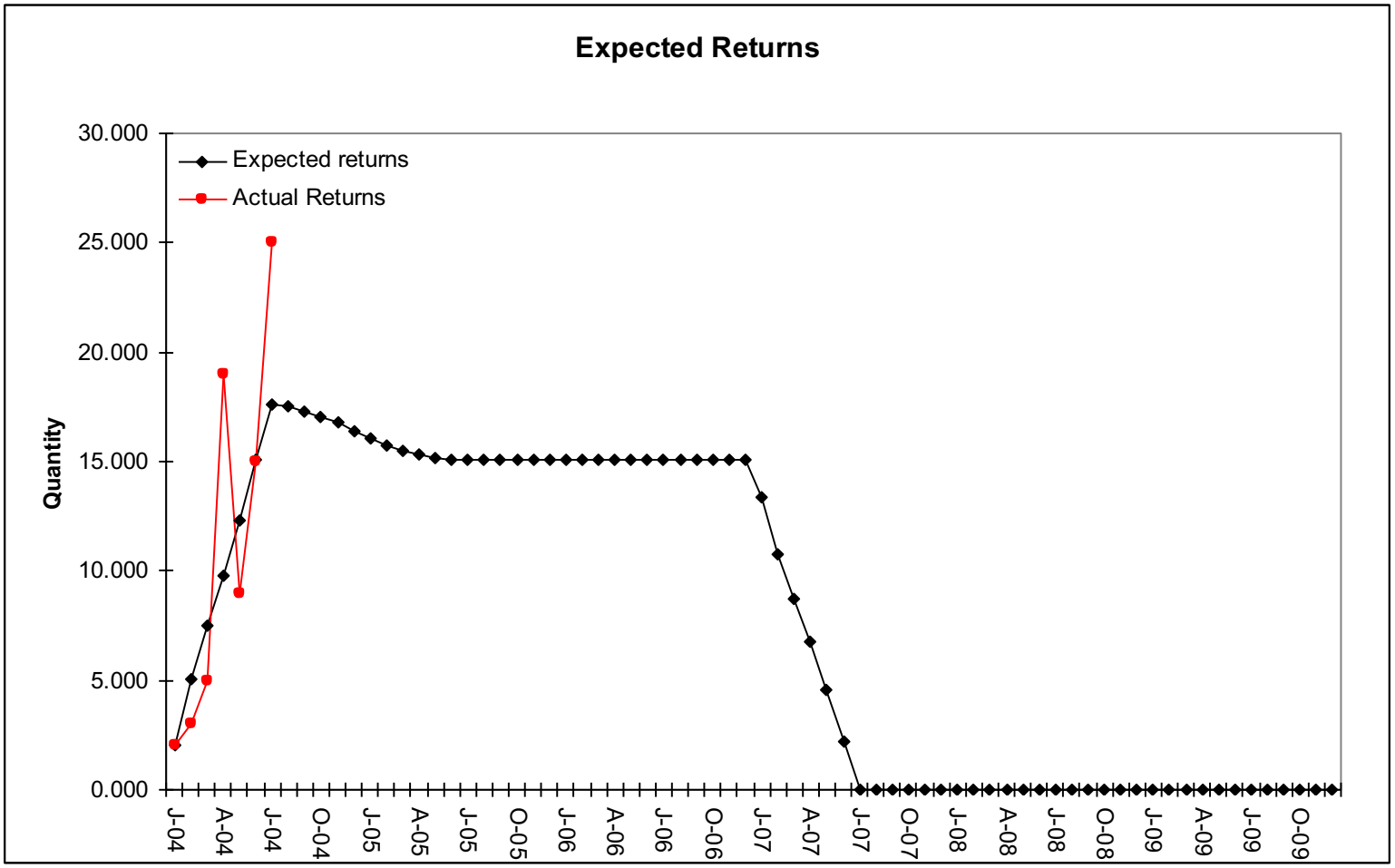
$$\hat{\lambda} = \frac{number\ of\ failures}{total\ unit\ test\ time}$$

Finding lambda (λ): The denominator is obtained by adding up all operating hours on test of all units tested, including those that failed and those that completed the test without failing. Example field time is determined by each months' sales multiplied by one. Each month's time is cumulative at flow points or intervals on the continuum and divided by returns for that month. This method provides an adequate piecewise exponential failure rate, which is required to use Poisson model for expected returns.

Product: NFS-6400 – cont'd

NFS-6400 – cont'd

Expected returns projected out over 36 months



Graph is from Haggerty's Excel Workbook

Note: This chart does not include replacement. Because the shipped quantity is much greater than returns, this is replacement is irrelevant.